Practice and thinking about teaching dynamic geometry course in Normal University

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Abstract: *According to the demand for modernization of education and the math education requirements of middle school, we open the dynamic geometry course in Huazhong Normal University. This paper records our experience of teaching dynamic geometry course and introduces the contents that can stimulate the interest of students or are very useful for students. The main achievements of students and the problems exposed in the teaching practice are also shared in this paper. At the end of the paper, the evaluation and feedback of the students to the course, as well as the expectation and prospect for the future are presented.*

1. Introduction

Dynamic geometry is a mathematical and graphic software technology developed rapidly in the last 20 years, whose active role in education has become a general consensus. According to the mathematics education requirements, China has successfully developed its own dynamic geometry software, i.e., Super-Smart-Platform (SSP).

Ref. [1] introduced the basic functions of the free version of SSP, as well as its applications in teaching. Ref. [2] discussed the mathematics teaching and learning value of setting up the dynamic geometry courses in normal university. Driven by these efforts, Huazhong Normal University and some other colleges began to open the dynamic geometry curriculum in 2007.

This course opens just once a week with 100 minutes (including 10-minute break), and there are about 60-80 students and one teacher in the class. Every student has a computer with SSP software installed, and the teaching by teacher and practice by students are performed alternately, each with half class time.

In the previous work [3], we have introduced the initial practice and reflection of setting up dynamic geometry course with SSP as the main software in some normal universities, as well as investigated the role of this course on the mathematics teacher education. Based on the previous work, this paper will introduce the experience and practice of opening dynamic geometry courses in Huazhong Normal University, and continue to explore the significance and modalities of setting up dynamic geometry course in normal university

2. Necessity of Setting Up This Course

In recent years, the employment situation for university students has been increasingly serious. More and more normal students graduated from normal universities as the result of increased college enrollment, and many non-teacher-oriented graduates also competed for the teacher job because of economic downturn, on the other hand, there are less and less jobs available in the middle schools. Therefore, the employment situation for normal students becomes more and more fierce.

Some people think that when employing new teachers, the middle schools should first consider the graduates from normal universities because they have been trained with professional teacher education. This remark sounds reasonable, but is also open to question. In fact, we believe that a student in normal university could be regarded as normal student only after he has mastered the basic skills for a normal student.

According to the investigation results, many higher normal universities do not take into account their own actual situation, blindly moving closer to the comprehensive university, as a result, the courses are not so reasonable. On the one hand, they provide "Education", "psychology" and other fundamental courses, but most students view these courses as "dogmatic theory", their interest in these courses is very low. On the other hand, the major courses set up in junior and senior year are rather difficult, such as "Real Variable", "Functional Analysis", "topology," "partial differential equations" and so on. These course are helpful to improve the students' mathematical self-cultivation, however, to study and understand the idea of these subjects, the students have to spend many time. Indeed, a lot of junior and senior students have not so much time and energy to study these courses well because of the employment pressures, for instance, they have to spend a lot of time preparing for the civil service exam, graduate entrance examination and so on. According to the survey results in Ref.[3], the normal students prefer those practical course similar to the dynamic geometry course, because the content of these courses can be of more direct use in the future teaching career.

With the rapid popularization of computers and information technology, there are growing needs for the young teachers with excellent education technology skills. Though the teachers in middle schools can learn some multi-media technology through in-service training or self-learning, their technical level is still low because the study time can not be guaranteed and some other reasons. Therefore, setting up the dynamic geometry course in university is also the demand of the job of a mathematic teacher. With more education technology knowledge and skills, these pre-service teachers can improve the overall level of teachers' technology knowledge skills in middle school when they start to work as teacher. At the same time, there is an interesting paradox: the in-service teachers know the importance of information technology skills but they have not enough time and chance, while the pre-service teachers have time and chance but they do not pay enough attention to study these skills.

In order to solve this problem, we take into account the dual role of normal students: college students and pre-service teachers, and focus on the two aspects of "interesting" and "useful" in teaching activities. The reason why the "interesting" placed in front is that interest is the best teacher. In the preparation of teaching materials [4], we have given special attention to the practicality and fun of the content.

3. What Can Inspire the Students' Interests?

Though university students are already adults, they still like to play for fun. In particular, some students feel that more than 10 years' school life before entering university is very hard, so they want to relax and make up for the lost playtime. In addition, due to various reasons, many university students do not like math, even for students majoring in math. The objective of setting up dynamic geometric course is not only to teach a software operation, but also hope to stimulate students' enthusiasm for learning math. If a normal student majoring in math do not like math, then after graduation, how can he be expected to stimulate his students' passion to learn mathematics?

The following are examples of dynamic geometry course that can arouse the interests of students to learn. It is noted that the interest talking about here refers to the happy discovery when the students explore the mathematical nature with SSP.

3.1 Case 1: Function Mapping

Many students feel that math is monotonous and boring. Prof. Chen Xingshen once said that math is full of fun, but most people find it is very difficult to understand that. Someone said: "God is a mathematician, and the only language that can describe the universe is mathematics!" For this sentence, whether a student or a teacher may not quite understand. How to use mathematical language to describe the universe? I am afraid it is difficult to answer. In the eyes of many people, the connection between mathematics and nature is not much. Mandelbrot, the well-known mathematician and founder of fractal geometry, once said: "Why is geometry often described as ruthless and boring? One of the reasons lies in its inability to describe clouds, mountains, coastline, or trees shape. Clouds are not spheres, mountains are not cones, coastlines are not circles, bark is not smooth, nor lightning spread along a straight line…. Mathematicians can not evade these questions nature raised. "

In fact, it does not need any advanced math, i.e., just the combination of some exponential function, sine function, cosine function and so on that we have learned in middle school can depict many objects in nature. Drawing the function curve of $\rho = e^{\sin\theta}\text{-}2\cos(4\theta)+\sin(\frac{2\theta\text{-}\pi}{24})^5$ on SSP, you may be surprised to see a butterfly. If you draw the function curve of $\rho = \frac{15}{\sqrt{17-16\sin\theta\left|\cos\theta\right|}}$, you will find that math is full with "Love" (Figure 3.1).

Figure 3.1 Butterfly and Love Pattern Resulted from Polar Coordinates

Polar coordinates are not very popular in middle schools. Many students feel that there already has Cartesian coordinate system, why introduce the Polar coordinates again? The two following examples are strong proof of necessity of Polar coordinates. The patterns are so beautiful and they are difficult to be obtained by Cartesian coordinate system. Of course, the Cartesian coordinate system has its own style. As shown in Figure 3.2, the patterns are drawn by simple linear equation and conic equation in Cartesian coordinate system.

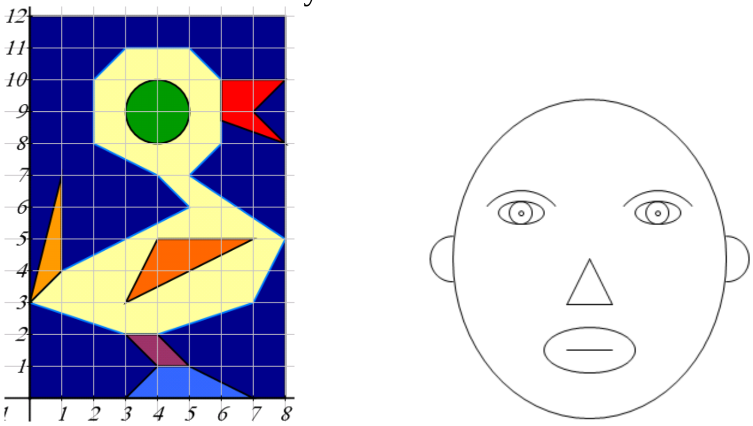
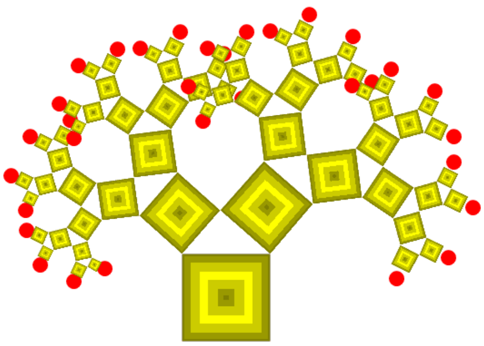


Figure 3.2 Patterns Resulted from Cartesian coordinate system

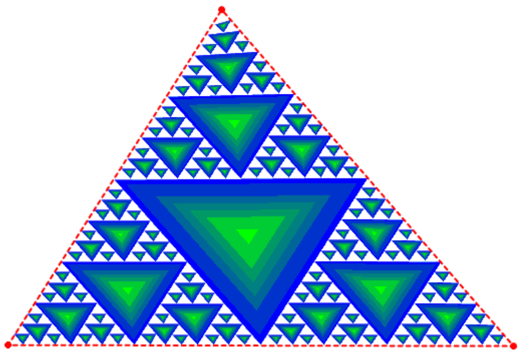
3.2 Case 2: Programming Mapping

It is laborious to draw a complex pattern by drawing a series of function curves one by one. However, computer is an indefatigable worker, it will implement the order given by people without complaint. The most problems the students majoring in math learn to solve when they learn programming are numerical calculation problems, such as determining prime number, finding a few daffodil numbers or dichotomy roots and so on, thus it is almost impossible for them to have any opportunity to feel the fun of computer mapping. While in the dynamic geometry courses, they can combine programming with mathematical mapping to create brilliant mathematical works. Of course, they should first learn the programming algorithm for the following patterns (Figure 3.3).

3, 4 and 5 are the most classic group of Pythagorean number. It is easy to associate the square of number with the shape of square. Drawing three outside square from the three sides of a right-angled triangle, respectively, then we can see that the area of the hypotenuse square is equal to the sum of the area of the two right-angle side square, which is called a Pythagorean diagram. Continuing to use the Pythagorean Theorem to decompose a square to be 2 square, 4 squares, 8 squares ..., a Pythagorean tree is resulted. Looking closely, you can find Pythagorean diagrams in the Pythagorean tree are similar, but the sizes are different. Based on their similarity, you can easily draw the Pythagorean tree by using iteration function of a computer, even make a dynamic effect.



(a)



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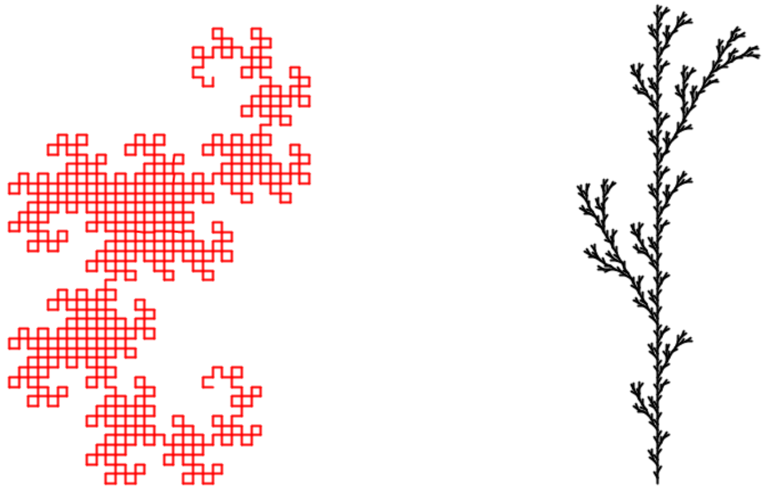


(c) (d)

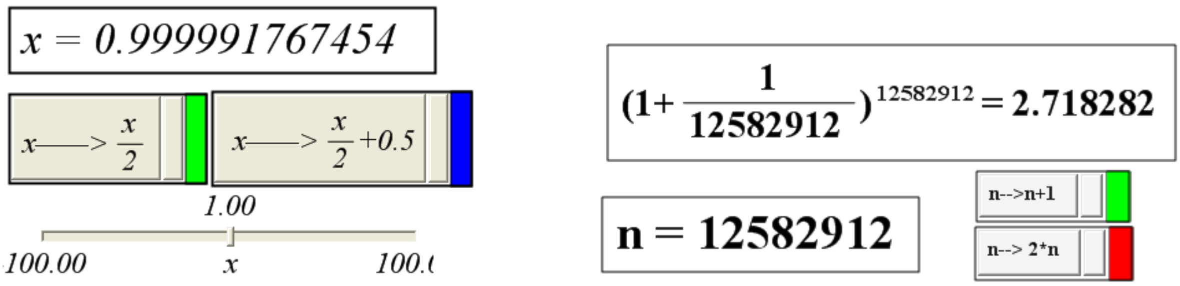
Figure 3.3 Patterns of Pythagorean Tree

3.3 Exploring the Limit by Animation

As for the fixed-point in mathematics, many people think it is very mysterious and very difficult. The following example can show you that there is a lot of fun with fixed-point. First, let's see a simple example: randomly generate a number, multiply it by 0.5 and obtain a new number, continue this process for this new number step by step, what is final number eventually?

Then we add a little difficulty to this problem: randomly generate a number, multiply it by 0.5 and then add 0.5 to get a new number, continue this process for this new number step by step, what is final number eventually?

It may be difficult for us to think out the answer, while a simple animation can demonstrate this process and show the answer. Let's measure parameters *x* whose initial value of -3 on SSP, make its animation with frequency 1, set the parameter range between *x* to *x**0.5 +0.5, and set the animation type to be a one-time movement, then we repeatedly click the button animation and find that the value of *x* approaches 1, as shown in Figure 3.4(a).



(a) (b)

Figure 3.4 Example of Limit

The benefit of a common law is that after grasping them we can solve a large class of problems. It is noted that animation has same benefit as common law. Figure 3.4(b) shows that after clicking the animation, n changes to $2n$, and the change trend of $(1+\frac{1}{n})^n$ can be illustrated. Similarly, we can easily explore the limits for other function such as $\lim_{x\to 0}\frac{\sin x}{x}$, $\lim_{n\to\infty}\sqrt[n]{n}$.

3.4 Case 4: From Connecting Line to the Envelope

This is a case for beginners, and even primary students can try this case. Connect the points on the lines with same color and number in Figure 3.5(a) into lines, and connect the points on the circle according to the number sequence into lines, then the result is shown in Figure3.5(b).

This case is very popular. It is simple and funny, and the cartoon images obtained can easier attract students. If some teachers want to reinforce the knowledge of points, they can change the connecting rules, for example, two points with the sum of 10 or product of 60 can be connected into lines. As shown in Figure 3.5(c), two points with the sum of coordinates of 13 are connected into lines. This kind of drawing is helpful to the students to learn the coordinate system. In this way, students may feel that coordinates is not something new, but just two straight lines perpendicular to each other with a number of scales, which makes drawing easier.

Are these investigations enough? No! After learning the inverse function, even after learning the hyperbolic, students may think more about Figure 3.5(c). They may guess that the curve surrounded by segments is hyperbolic. In fact, it is not hyperbolic, but a parabola. As shown in Figure 3.5(d), generate two points D and E on the two sides of angle ABC respectively, and the length of them meet the requirement $\frac{AD}{AB}=\frac{BE}{BC}$. When point D moves on line AB, the graph enclosed by the locus of DE is a parabola.

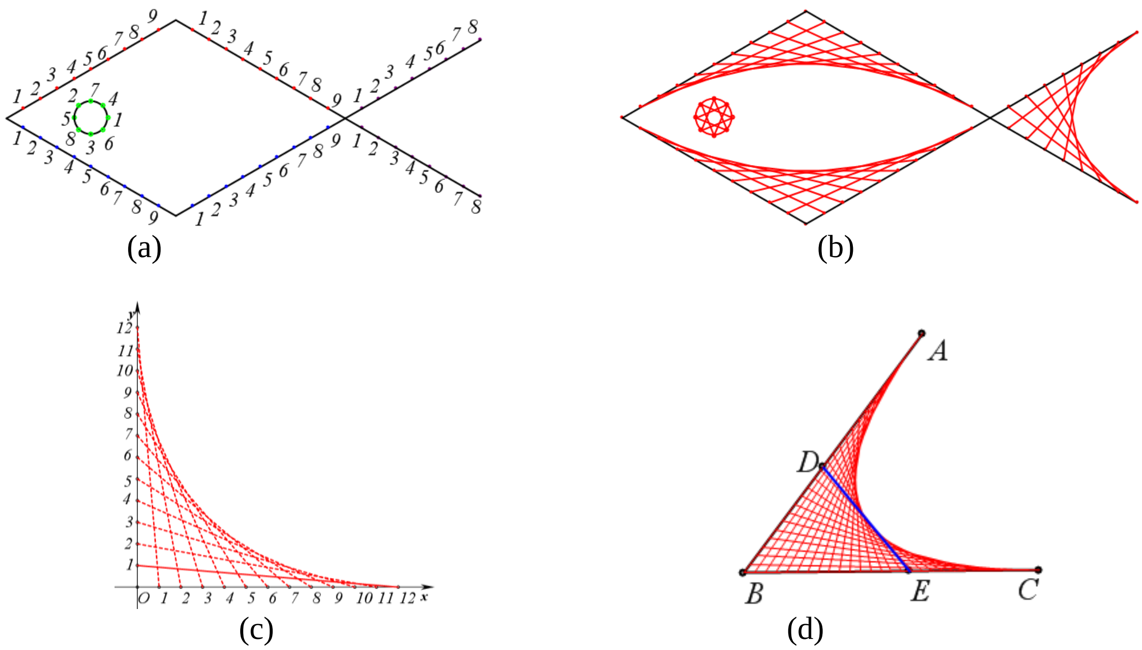
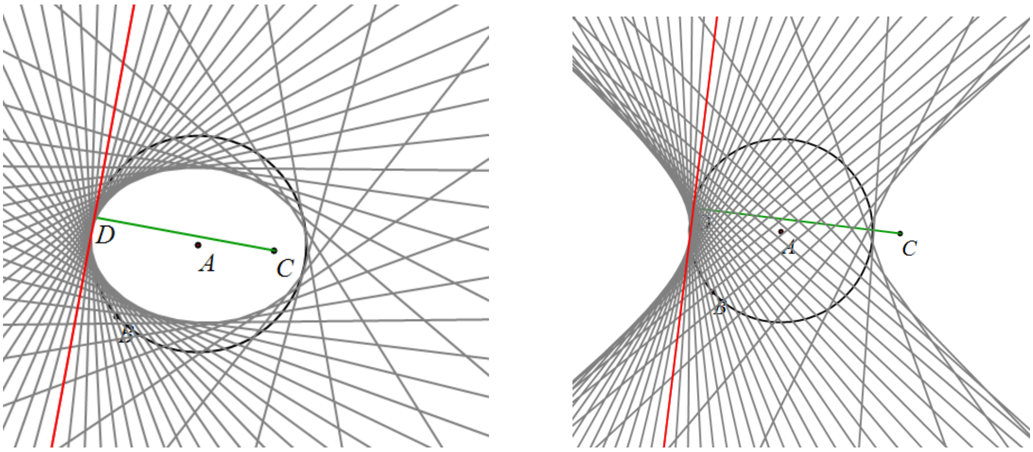


Figure 3.5 Illustrations of Connecting Points into Lines

Can we obtain an ellipse or hyperbola by the same way? Sure! As shown in Figure 3.6(a), point C is a point inside a circle and point D is on the circle, and the red line is the perpendicular line of segment CD. If point D moves round the circle, the envelope formed by the perpendicular line is an ellipse. Conversely, if point C is outside of the circle, the envelope formed by the perpendicular line will be a hyperbola, as shown in Figure 3.6(b).



(a) (b)

Figure 3.6 Ellipse and hyperbola obtained by the envelope

Many students majoring in math are confused by various abstract concepts (such as the envelope in differential equations), it is a pity that they do not know they have already seen these shape. The setting up of dynamic geometry course reduces the students' regret and help the students learn mathematics when playing with this software. Obviously, dynamic geometry is so powerful that the simple connecting problems in primary school can be associated with the envelope of differential equations in college level by the dynamic geometry software.

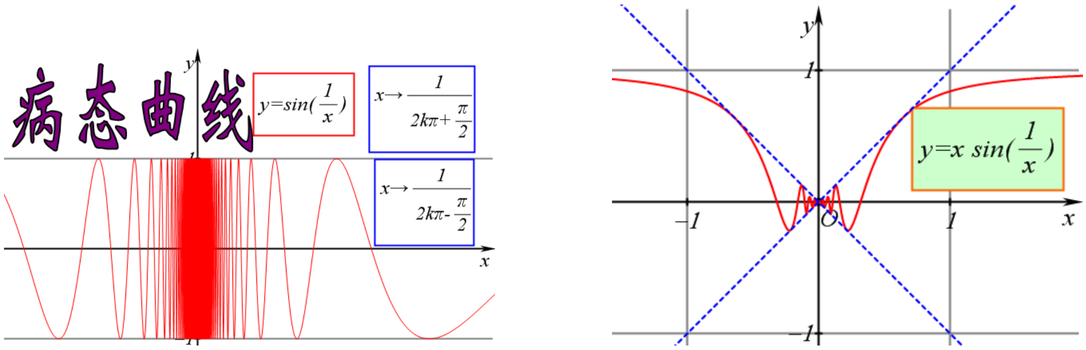
4. What the Students Found Useful

Being interesting is not enough for setting up a course. The opening of any course has its own purpose. Of course, the function of stimulating students' interest in mathematics mentioned above is certainly one of the roles. The following will further introduce the role of this course. We first introduce its role in higher mathematics, and then introduce its role in mathematics teaching of middle school.

4.1 Case 5: Understanding Special Functions

We usually encounter pathological function in higher mathematics, which have some strange features. For example, when $x \to 0$ in function $y = \sin(1/x)$, the function will vibrate with increasing frequency, which is so difficult to draw that few textbooks have any graph of this kind of functions. For such a special function, students would be interested to see how it looks like. For another function $y = x\sin(1/x)$, which just has one more item x, is not considered to be a pathological curve, why?

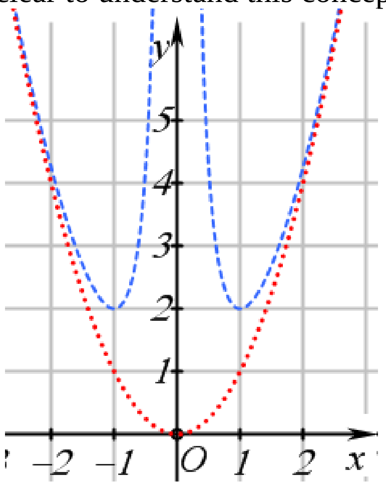
Students will be able to draw the function curve of these two functions after they have learned the dynamic geometry course. Figure 4.1(a) demonstrates that the strong vibration of $y = \sin(1/x)$, and there is no strange vibration phenomenon in Figure 4.1(b), which illustrates the function curve of $y = x\sin(1/x)$



(a) (b)

Figure 4.1 Illustrations of Pathological Function

A student said: when I met the concept of asymptotic curve, I did not understand, such as the asymptotic curve of $y = x^2 + \dfrac{1}{x^2}$ is $y = x^2$; now by drawing the function curve through SSP (Figure 4.2), it is clear to understand this concept.



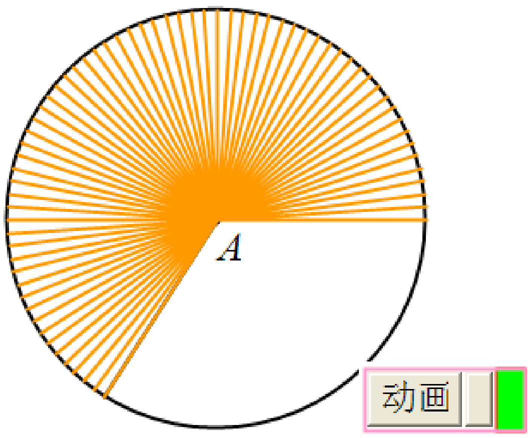
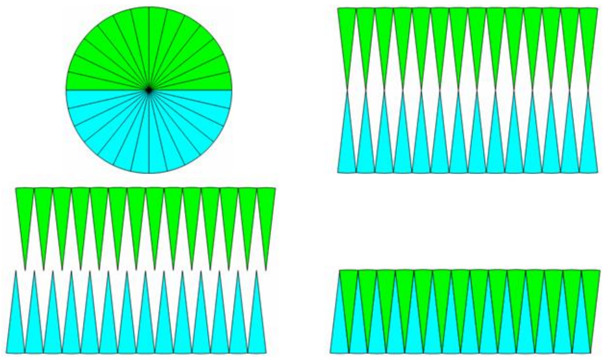


Figure 4.2 Illustrations of Asymptotic Curve **Figure 4.3** Circle obtained by cycling a line

4.2 Case 6: Understanding calculus: from line to circle surface

There are many way to draw a circle surface. For example, if we drag a segment around one of its two end points, a circle surface can be resulted, as shown in Figure 4.3. Figure 4.4 gives another way to get a circle surface. As can be seen, a circle surface can be divided into many small fans, and the fans can be pieced together into a parallelogram. If the fans are small enough, the parallelogram can be a rectangle. It is clear that the reverse process is a new way to draw a circle surface, showing the idea of calculus.

Circle surface can also be viewed as the result of a set of homocentric circles, whose radius range from 0 to R, as shown in Figure 4.5. From Figure 4.6 we can see the circle surface can be outspread to a triangle with area πR^2.



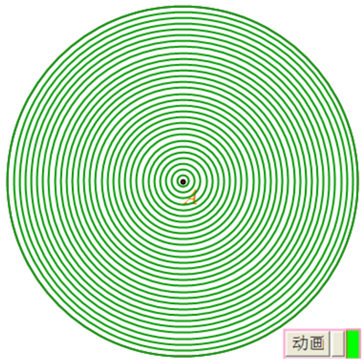


Figure 4.4 From circle to parallelogram **Figure 4.5** Circle obtained by cycling a line

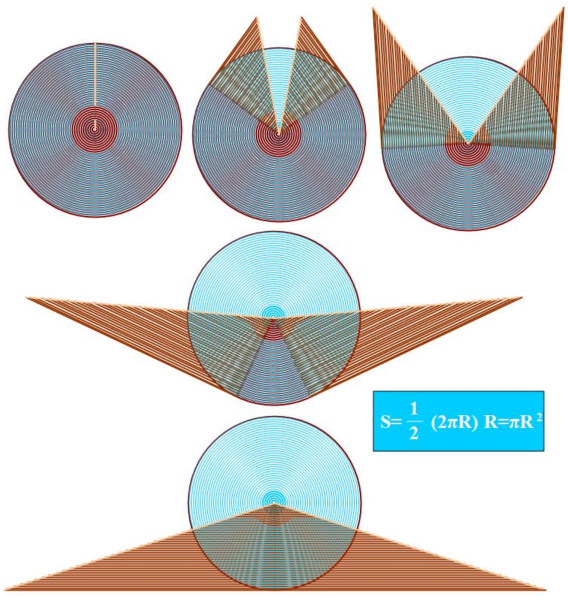
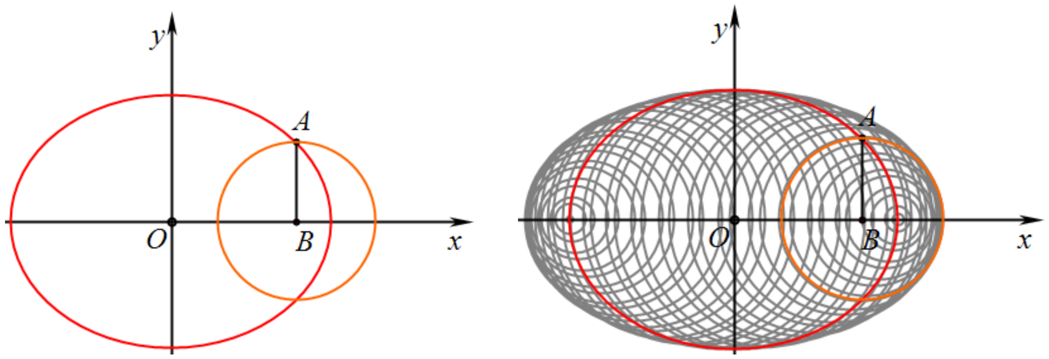


Figure 4.6 Outspread a circle surface into a triangle

4.4 Case 7: Seeing is believing

When learning the knowledge point of envelope, some students associated envelope with tangent. They think that there will exist tangent relationship if there is envelope. Here we give a counter-example to prove this view is incorrect. Point A is a random point on ellipse in Figure 4.7(a), and point B is on the long axis and segment AB is perpendicular to the long axis, then we draw a circle with center point B and radius AB. When point A moves round the ellipse, the envelope of the locus is shown in Figure 4.7(b). It is easy to see that this circle is not tangent with the elliptical envelope when point A approached the end points of long axis..



(a) (b)

Figure4.7 Envelope of locus of moving circles

With SSP, it is unnecessary to repeat some cumbersome experiments any more when learning probability. Figure 4.8 is a very classic probability experiment designed by British scientist Francis Galton. There are several rows of nails and several containers in this experiment, and a ball is allowed to fall from above the top nail. When the ball hit a nail, it will fall to the left side of the nail or the right side of the nail with same probability. Row by row, it will fall into a container. This animation is an interesting experiment to demonstrate probability.

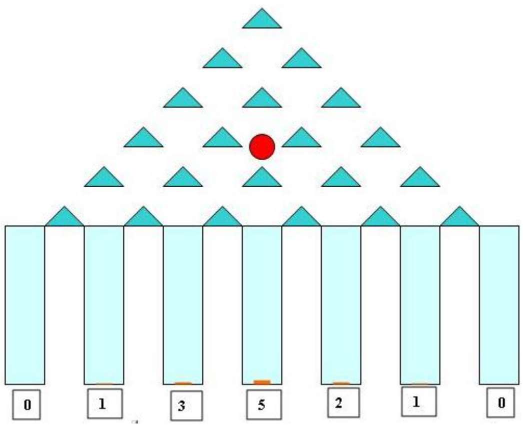


Figure4.8 Galton Board Experiment

4.5 Case 8: Universal Proof of the Pythagorean Theorem

Gou-gu theorem is known as the Pythagorean Theorem in the West. It is said that the Ancient Greek mathematician Pythagoras discovered the relationship of the length of the three sides of right-angle triangle by observing the floor pattern. At that time, the floor pattern is the square pattern with same size, so the Pythagorean Theorem was first discovered for this particular case of isosceles right triangle.

There is a floor pattern shown in Figure 4.9. Obviously, this floor is paved with two different sized squares. If you can tell that this figure contains three types of proof of the Pythagorean Theorem, then you are so lucky to have a good geometric intuition. In fact, there is more mystery about this figure. If we device a square cardboard (the square of side length of which is equal to the sum of squares of the side length of the two square of floor) and throw it to the floor randomly, then we get a proof. If the location is a special position, the cutting of the square will be relatively simple; otherwise it will need a little more effort.

In order to demonstrate this process more clearly, we make a courseware called “universal cutting of the Pythagorean Theorem”. Simple as it is, this courseware is very powerful. Just by dragging the two points on the figure, we can generate various proofs to prove the Pythagorean Theorem.

Figure 4.10 only illustrates two patterns of the proofs. You can find more as long as you have time. It is apparent that generating a large number of proofs is an easy and interesting thing by computer software, which is rarely seen before. This kind of courseware is very necessary in teaching and learning.

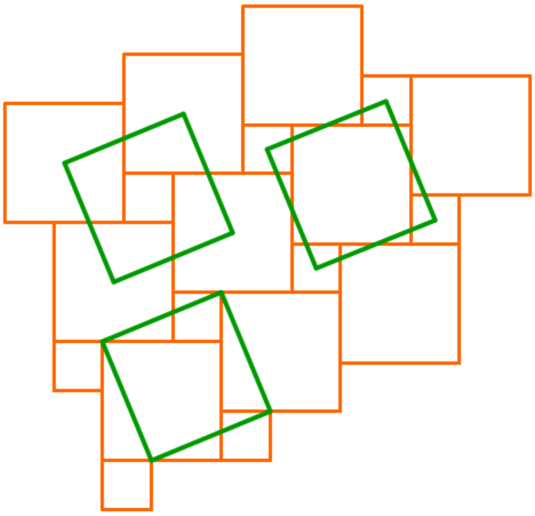


Figure 4.9 Illustration of proof of the Pythagorean Theorem

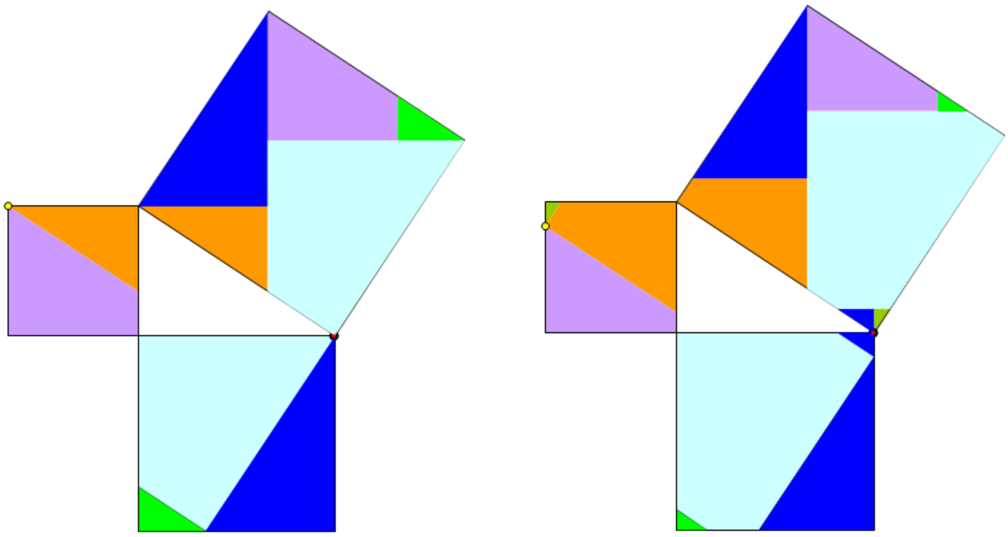


Figure 4.10 Two Patterns of universal cutting of the Pythagorean Theorem

4.6 Case 9: College Entrance Examination Questions and the Hula Hoop

Information technology is not only used in a new lesson to stimulate interest and solve the teaching difficulties, but also useful and helpful in exercise lessons. The following is the records of a teacher.

"Here is a problem: point $P \in \{(x, y) \mid (x\text{-}2\cos t)^2 + (y\text{-}2\sin t)^2 = 16 (t \in R)\}$, then the area of the graph enclosed by all the P which satisfy the conditions is ______.
How can I make the solution of this problem understood? When I came back to office, one old teacher told me that the graph formed by all P is like the figure formed by a rotating hula hoop. Great! Then when I explained this problem to the second class, I add this explanation: in a visual way, the figure formed by point P is like the locus of a hula hoop when we exercise with it. The circle center O is like our waist, when the hula hoop touches our waist, it is the position of circle O_1. Since hula hoop can not cross the waist, so the inner circle should be remove."

This problem is a little difficult for middle-school students, requiring strong imagination. For teachers, it is also not easy because it involves the moves and changes of figure. The solution to this problem can be attributed to the direction of the old experienced teacher. The metaphor compared this solution to a hula hoop plays a key role in this process, which reflects the accumulation of many years of experience. However, young teachers always do not have so much experience, is there any shortcut solution? Certainly, using information technology is a good solution. After drawing the function curve of $(x\text{-}2\cos t)^2 + (y\text{-}2\sin t)^2 = 16$ and tracking the circle, we make the animation of parameter t, and then we can get the graph shown in Figure 4.11. This process can be completed in just several minutes by SSP. However, many teachers and pre-service teachers find it difficult to solve this problem, which shows the necessity of using information technology in teaching.

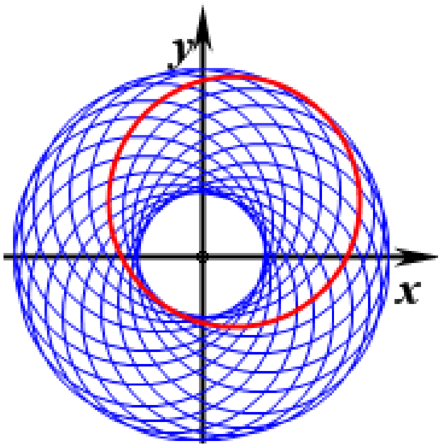


Figure 4.11 Comparison of Hula Hoop with the Solution to a Problem

4.7 Case 10: Example of exploration of elementary mathematics

Ptolemy theory is a famous theory, the content of which is that in a quadrilateral whose four vertexes are on a circle, the product of two diagonal lines are equal to the sum of products of two pairs of opposite edges. If the point D expands into a circle (Figure 4.12) and the three sides are replaced by the corresponding tangents, then ask whether the equation is still valid?

By using the measuring function of SSP, we can confirm this lemma by calculation, as shown in the left of Figure 4.12. Can you prove this lemma?

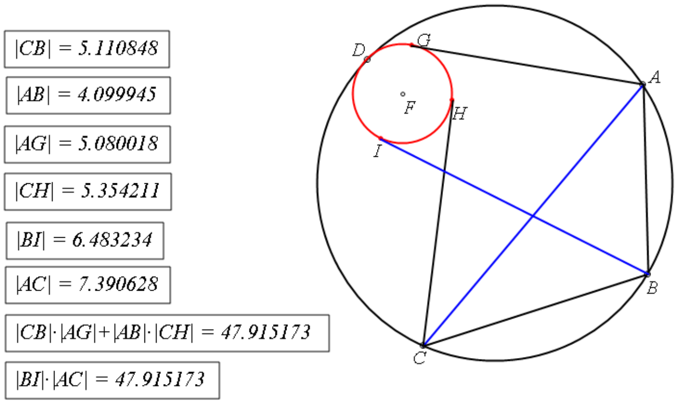
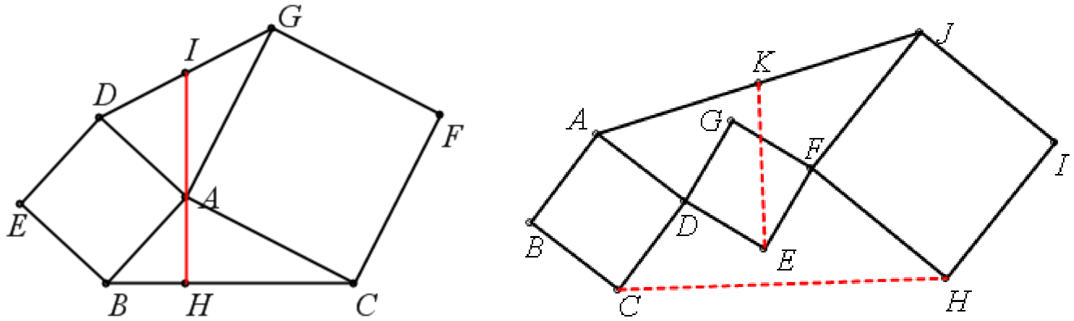


Figure 4.12 Expansion of Ptolemy theory

In fact, point not only can expand into a circle but also can be expanded into a square. For example, a geometry proving problem can be extended as follows:

The initial problem (Figure 4.13(a)) is: draw two squares with its side be the side of triangle ABC, respectively, point I is the midpoint of segment DG, and H is the intersection point of line BC and line IA, then prove $AI \perp BC$ and $BC = 2AI$.

This problem can be extended to a new problem by expending a point to a square, as shown in Figure 4.13(b), where square ABCD, square DEFG and square FHIJ have intersection point D and F, and point K is the midpoint of segment AJ, and then proves $EK \perp HC$ and $HC = 2EK$.



(a) (b)

Figure 4.13 Expansion of a point to a square

5. The Teaching of Conic Curve

The course of dynamic geometry not only equips the students with a computer tool, but also gives the students a chance to re-understand mathematics and get rid of the former misconception that math is equal to problem solving. In order to obtain good teaching effects, we spend a lot of time collecting material and preparing courseware. At the same time, we collected our reflection on the teaching process. The following introduces the author's teaching process for the part of conic curve.

5.1 From the Vertical Line Segment to the Nested Intervals Theorem

First, let us think about this problem: find the minimum distance from line *MN* to a point *O* outside the line? This problem is so easy that you can make a vertical segment and measure it. However, if you do not continue further you will miss the beauty. Let us go on to ask how to make vertical line segment. Usually, we draw a vertical segment by the ruler and compass. As shown in Figure 5.1(a), we first draw a circle with center on point *O*, and the intersection point of the circle and line *MN* is M_1 and N_1. Similarly, the next step is drawing a little smaller circle with center on point *O*, and intersection point of the circle and line *MN* is M_2 and N_2. Apparently, the midpoint of M_1N_1 is also the midpoint of M_2N_2, but the length of latter is smaller then that of former. Continuing this process until point M_i and N_i coincides, as shown in Figure 5.1(b), we can see the resulted circle is tangent with line *MN*, and the length OM_i is what we seek.

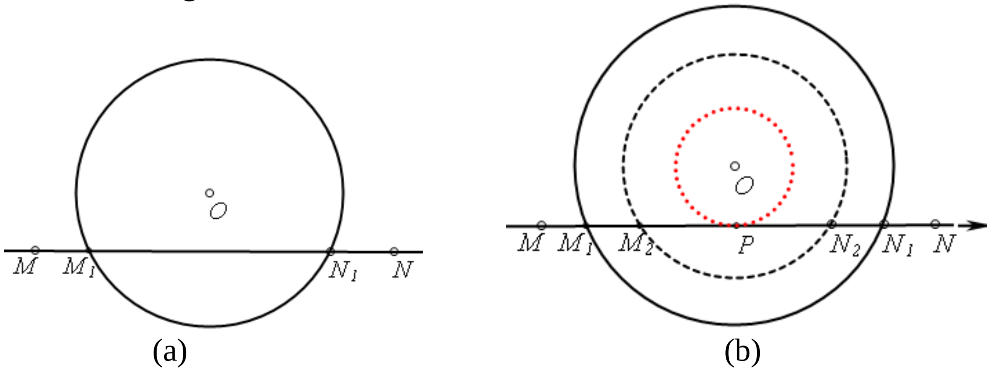


Figure 5.1 New way to draw the perpendicular bisector

Some people may say that it is too troublesome to get this solution compared with the usual way of drawing vertical line. However, strategically detour is to move forward fast. After careful analysis, we can find here we use the mathematical method of dynamic approximation, more importantly, which contains nested intervals theorem in mathematical analysis. Nested intervals theorem is rather abstract for the college students who are new to higher mathematics. Figure 5.1(b) can be regarded as an intuitive model of nested intervals theorem, the point *P* on which is the only point ξ in nested intervals theorem.

If a closed interval column has the following properties: **1)** $[a_n,b_n]\supset[a_{n+1},b_{n+1}]$, $n=1,2,\cdots$; **2)** $\lim_{n\to\infty}(b_n-a_n)=0$; then we call $\{[a_n,b_n]\}$ a closed nested interval, or nested interval for brevity. Obviously, if $\{[a_n,b_n]\}$ is a nested interval, then $a_1\le a_2\le\cdots\le a_n\le\cdots\le b_n\le\cdots\le b_2\le b_1$.

Nested intervals theorem: If $\{[a_n,b_n]\}$ is a nested interval, then there exists a unique real number $\boldsymbol{\xi\in[a_n,b_n]}$, i.e., $\boldsymbol{a_n\le\xi\le b_n}$, $n=1,2,\cdots$. This theorem is illustrated in Figure 5.2.

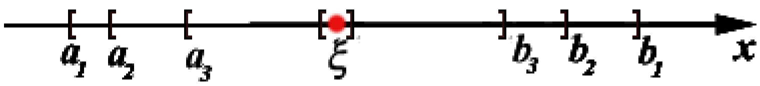


Figure 5.2 Illustration of Nested Intervals Theorem

In recent years, mathematics education researchers have repeatedly stressed the infiltration of higher mathematics in elementary mathematics, while the content of elementary mathematics is rarely seen in higher mathematics teaching, which confuses many college students because the elementary mathematics knowledge accumulated so many years seems useless when they start to learn higher mathematics. In fact, the visual models of many higher mathematics theorems can be found in elementary mathematics.

5.2 From Circle to Ellipse

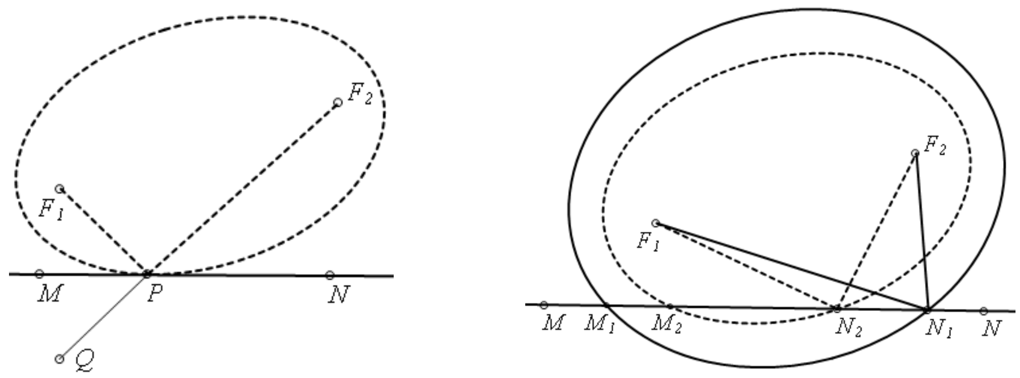
Here is a problem: point F_1 and F_2 are at the same side of line *MN*, the problem is to ask how to find a point *P* on line *MN* that makes $F_1P + PF_2$ minimum?

This problem is a little more difficult, which is a classic problem in plane geometry. Since it is related to a general from Ancient Greek, it is also known as the "general drinking the horse problem". As shown in Figure 5.3(a), drawing the symmetry point of F_1, i.e., point *Q*, with line *MN* being the axis of symmetry, connecting two points F_2 and *Q*, the intersection point of which and line *MN* is point *P*, which is what we seek.

The difference between these two problems can be considered as the distinction between one point and two point, which makes it easier to associate with the relationship between ellipse and circle. Through previous exploration, we can solve the "general drinking the horse problem" with an ellipse instead of circle. As shown in Figure 5.3(b), draw an ellipse with two fixed-point F_1 and F_2 as focal point, the intersection points of which and line *MN* are M_1 and N_1, respectively. Then draw a smaller ellipse with the same focal point on F_1 and F_2, which intersect line *MN* on the points of M_2 and N_2. Obviously, $F_1N_1 + F_2N_1 > F_1N_2 + F_2N_2$. This process continues until M_i and N_i coincides, then the resulted ellipse is tangent with line *MN*, and $F_1M_i + F_2N_i$ is the minimum distance.

If we put the two approaches together, as shown in Figure 5.3(b), we can get a optical properties of ellipse: if the light radiates from a focal point, then according to the law of reflection that the angle of incidence is equal to the angle of reflection, all the light will focus on the other focal point after the surface reflection of ellipse mirror. In other words, the two acute angles formed by the tangent line and two radius across the tangent point are equal.

The resulted conclusion here is quite natural, intuitive and impressive, and it do not need any calculations. For a long time, there seems to be only one solution to the "general drinking the horse problem", which is mainly due to the hard process of drawing ellipses by hand. Now, with the ellipse drawing function in dynamic geometry technology, more approaches can be used to solve problems. It is natural to think that how many more problems can be improved by traditional means?



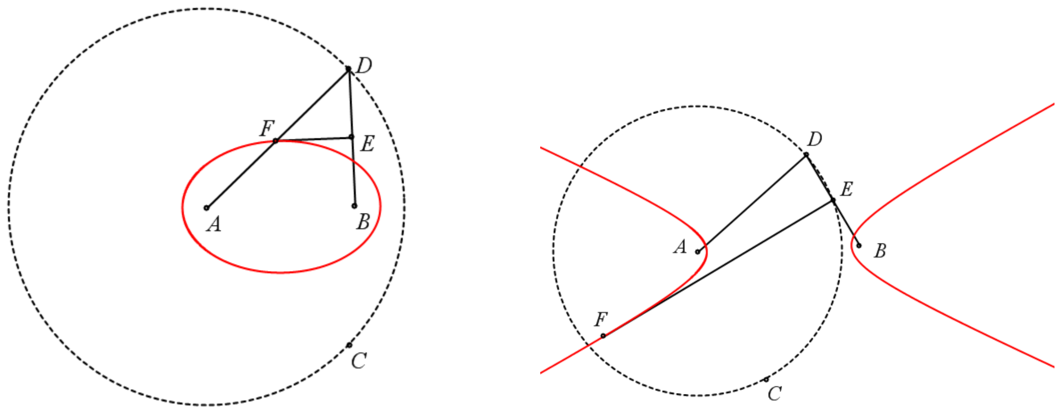
(a) (b)

Figure 5.3 Two Ways to Solve "General Drinking The Horse Problem"

5.3 From Two-combined-into-one to One-divided-into-two

The "general drinking the horse problem" deals with the summation of two segments. The solution is to transform two noncollinear segments into a collinear segment by making a symmetry point. On the other hand, we need to cut one segment into two parts for some problems, such as drawing ellipse.

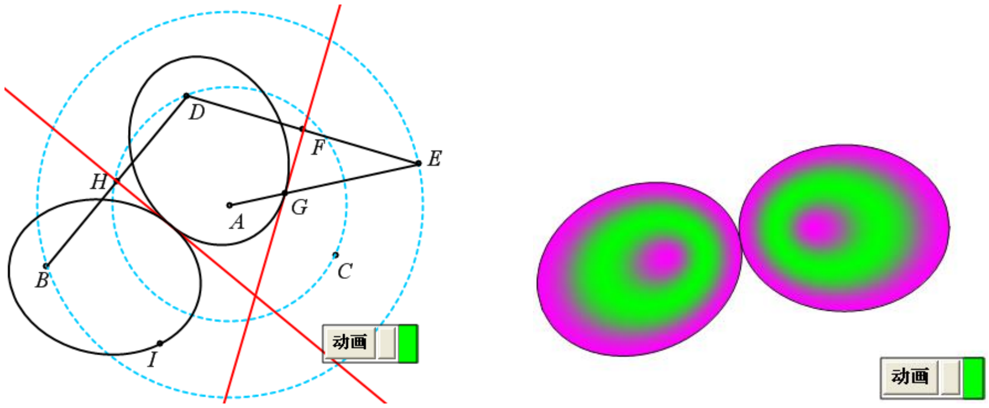
Considering drawing an eclipse by the definition, we draw point *A* and *B* as focal points, and search for a fixed length. Since only the definition of circle has something related to fixed length, we then draw a circle with point *A* as center and *AC* as radius, now *AD* is the fixed length. From a new angle to see Figure 5.3(a), *MN* can be seen as the perpendicular bisector of F_1Q and F_2Q can be divided into F_1P and F_2P. We can use SSP to probe into this issue. As illustrated in Figure 5.4(a), connect point *B* and *D* and then make the perpendicular bisector of *BD*, which intersect *AD* with *F*. The locus of point *F* is an ellipse while point *D* is moving on the circle. The benefit of information technology and dynamic geometry is that some elements in graphics can move and be dragged and then some implicit rules can be detected. The locus of point *F* is a hyperbola if point *B* is dragged out of the circle, as shown in Figure 5.4(b). The reason is simple because the difference of *FD* and *FA* is fixed.



(a) (b)

Figure 5.4 New Ways to Draw Eclipse and Hyperbola

Based on Figure 5.4(a), we can easily draw two tangent ellipses, as shown in Figure 5.5(a). If D moves round the circle, the tangent relationship is unchanged, as shown in Figure 5.5(b), where the two ellipses are filled with colors.



(a) (b)

Figure 5.5 Two tangent ellipses

5.4 Reactions of Students

Students approve this mode of teaching. They say that although it's a short teaching segment, it embodies extensive connection, smart analogy and intuitionistic model. Using this method, many classical questions can be illustrated in a new, simple but interesting way.

6. The Students' Appraisement And Feedback of This Course

Student Lai: There is some reason why this course is called Dynamic Geometry. On seeing this title, I'm curious. How can the geometry graphics be dynamic? This is fantastic! I attended this course with curiosity, which was soon satisfied by the colorful and funny class. I can still remember the lingering of the "double eggs depending on each other", the delight of the "four tortoises chasing each other" and the magic of the "puzzles with Pythagorean theorem". All these dynamic graphics impressed me so deeply that I will not forget this course in my life. The SSP is so powerful that I will keep it in my computer forever!

Student Nong: My knowledge was increased after attending the course of SSP. Learning and using this software improve my teaching wisdom, provide chance for me to give full play to my talent and individuality in my future teaching, and will surely assist me in my future career. Although this course is over, I will continue to learn to use this software to acquire more knowledge.

Student Xu: My interest of learning this course is strong because it's so interesting. To be frank, I have never missed any class or done anything unrelated in the class. Visualized and intuitive, this course can easily trigger the enthusiasm of the students' learning of math and help teachers demonstrate the knowledge which is unable or difficult to be described by words. This software can aid me to explore and research. During the problem solving process, it can help me gain the accurate result quickly.

Student Yuan: The supreme merit of this software is its practicality. It has a strong capability of graphics drawing, and also offers the programming environment. Various aspects of problems such as analytic geometry, function graphics, plane geometry, algebra operation, solid geometry and even some problems in other subjects can be solved by it. The plane locus and solid geometry are the most difficulty parts in high schools because they require strong abstract thinking ability, which is also the main reason why many students lose the interest of math. For many who do well in these courses, they may not understand the principles completely, merely tackling the problems by experience or certain fixed approaches. How can they be expected to master the knowledge and then apply it at will in the practice? So, it's quite necessary to introduce the SSP to the primary and middle school teaching and through this way, the transform of teaching style may be brought about. I think that as a normal student, it's a necessity learn and use SSP well.

Student Zhahan: I really didn't know anything about this course when I select it. I merely wanted to acquire one more credit. Later I knew this software was used to make courseware. So I wondered whether this software was the same as those usually used software such as PPT, Flash? To my surprise, the SSP is not boring and tedious in the least and more friendly in use. I once explained a problem to my classmates many times but in vain. Amazingly, when I tried to use the SSP to illustrate, all of them understand immediately. The biggest gain of attending this course is not only I acquired a new skill but also I could tackle the difficulty of drawing complex function graphics by hand. I will study harder and learn more about this software in my future.

Student Gan: I think this course is the most valuable one in my university life. I'm not flattering but telling the truth. I am not ambitious, what I want to be is just a math teacher in high school. Many courses in my major are quite difficult, such as "real variable functions", which is rather obscure for me. We have to recite the answers before examines, thus studying turned to be tedious and meaningless. I can't obtain any happiness in the pile of figures and letters but at least, the course of Dynamic Geometry let me have some fun and satisfaction from math, which I have not tasted for a long time.

7. The Problems of the Students Exposed in This Course

The problems that the beginners proposed are mainly due to their unfamiliarity with this software. Once they are more skilled, the most encountered problem is resulted from their weak foundation of math. Here give two examples:

Problem 1: In the part of learning integral division of function curve and measurement of summation of integral division (Figure 7.1), many students wonder why the numerical integration remains the same while the other two integral value change with the division number *n*?

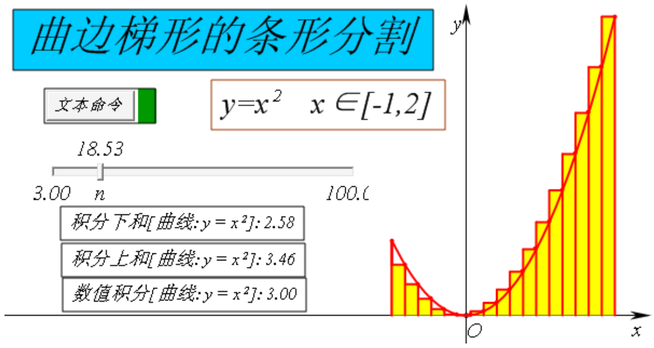


Figure7.1 Integral Division of Function Curve

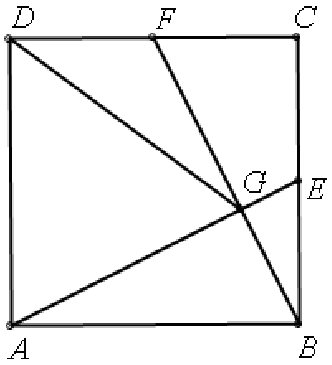


Figure7.2 Demonstrate of a Example

Problem 2: This is a question in a final examination, and many students failed to solve it. The problem is: In Figure 7.2, point E and F are the midpoints of BC and CD respectively. They intersect at point G and $\angle ADG = 50^{\circ}$, then what's the angle of $\angle BAE$? Please give your opinion of this question.

Unfortunately, many students didn't detect that the given data is wrong.

8 Conclusions and Prospects

Based on the practice of opening dynamic geometry courses, the following conclusions can be reached:

1) Students enjoy learning dynamic geometry;
2) Learning dynamic geometry can foster and boost the students' interests to mathematics and computer;
3) It can help the pre-service math teachers to be better teachers if they grasp the knowledge and skill of dynamic geometry;
4) The reasoning and operating capability of students can be promoted by learning dynamic geometry;
5) It's much easier to detect the deficiency of students' math ability during the course of dynamic geometry teaching and learning.

Geometry was once a crucial course of cultivating and training the thinking habit and ability of the youth in ancient Greece. Today, the classical geometry courses have already withered away. Although the dynamic geometry course may be an advisable substitute, there are still some difficulties as follows:

1) Since there are so many aspects involved in the dynamic geometry course such as overall introduction of the software, training how to use the software to solve problems, do research, write papers etc., the credit hour are not enough;
2) There are not enough teachers and classrooms. If more students take the dynamic geometry as an elective course, there will be in short of teachers and classrooms.
3) There should be corresponding teaching resources for students with different background of major.

To solve the problems above, we propose that the course of math teaching and dynamic geometry should be banded together. For example, if the courses such as “elementary mathematics research” and “math competition in high school” can be combined with dynamic geometry, it will help to improve the training quality of students in normal schools. On the other hand, setting up online dynamic geometry course can contribute to the problem solving of the deficiency of teachers and facilities.

At present, there have been already several universities that offer the dynamic geometry course. It’s believed that there will be more and more normal universities to offer this course, and set it as a compulsory course for the students majoring in math and information technology when it’s possible.

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Supplementary Electronic Materials

Video Clips for Figures 3.1, 3.1b, 3.2a, 3.2b, 3.3a, 3.3b, 3.4a, 3.4b, 3.5 and 3.6.
Video Clips for Figures 4.6, 4.9, 4.10 and 4.11.
Video Clips for Figures 5.1, 5.3, 5.4 and 5.5.

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